

# Modeling and solving a logging camp location problem

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**Abstract** Harvesting plans for Canadian logging companies tend to cover wider territories than before. Long transportation distances for the workers involved in logging activities have thus become a significant issue. Often, cities or villages to accommodate the workers are far away. A common practice is thus to construct camps close to the logging regions, containing the complete infrastructure to host the workers. The problem studied in this paper consists in finding the optimal number, location and size of logging camps. We investigate the relevance and advantages of constructing additional camps, as well as expanding and relocating existing ones, since the harvest areas change over time. We model this problem as an extension of the Capacitated Facility Location Problem. Economies of scale are included on several levels of the cost structure. We also consider temporary closing of facility parts and particular capacity constraints that involve integer rounding on the left hand side. Results for real-world data and for a large set of randomly generated instances are presented.

**Keywords** Logging camps · Capacitated facility location problem · Mixed integer programming

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## 1 Introduction

### 1.1 Context and scope

*Context* Log harvest planning in the forestry sector has changed throughout the last decades. Both silviculture and harvesting in Canada have become more sophisticated and now pose complex planning problems to get the most from the available regions and harvest cycles. Based on a wide variety of considerations, a long-term plan is designed to determine the volume and regions for wood logging. These decisions are commonly divided into smaller time periods, as logging activities and road construction within a single logging region typically take several months.

Due to political and environmental issues, as well as the size of the country, harvesting plans tend to cover wider territories than they used to. Often, sparse logging is necessary to certify the forestry operations. Several questions arise such as the location and capacity for administrative services, sorting yards and central log processing stations. Similarly, the location where the workers involved in forestry activities are accommodated gains in importance. If villages or cities are close, workers can be hosted at their homes or at motels. However, logging regions in Canada are often widely distributed and located far from such hosting options. In that case, accommodating the workers in the closest village or city is rarely an attractive option, as the commuting time and transportation costs are too high. Transportation times would consume a significant portion of the potential productive time. Furthermore, an additional salary is commonly paid when the transportation times exceed a certain threshold.

A common solution to this problem is the construction of logging camps in which the workers are accommodated. Logging camps are typically located close to the logging regions so that the transportation costs for the workers are reasonable. When allocating each work crew to a camp, the accommodation costs are given as a cost per day per worker. In order to host all workers, the construction of new accommodations may be necessary. The larger a camp, the smaller the daily cost per person. Hence, a small number of large camps results in smaller accommodation costs than a large number of small camps. However, the fewer camps are available, the higher the transportation costs tend to be, because their location is less flexible. The construction of a new camp or the relocation of an existing one may pay off in the long term as the traveling costs to the logging regions may be much lower.

*Scope* This work investigates the possibility of constructing and relocating camps for the accommodation of workers, considering the harvest planning for the next five years. The problem is motivated by the needs of a Canadian logging company. It consists in finding the number of camps that have to be constructed or relocated, their size and their location such that the total costs for accommodation and transportation are minimized. The interesting question is whether such an investment in camp construction and relocation pays off, considering the operational logging and road construction planning for the next five years. It is important to note that the actual work crew assignment between accommodations and work regions is not relevant in practice. It is only used to determine the minimum capacity level necessary to host all workers. For the operational work crew assignment, other planning tools will be used. It is assumed that all information about work crews, logging regions and distances are known at the beginning of the planning and are not subject to uncertainty.

### 1.2 Contributions and organization of the paper

*Contributions* Due to the complexity of the problem, manual planning approaches usually do not yield optimal solutions. The main objective of this paper is to propose a formulation

for the problem that can be solved by a general-purpose solver for instances of reasonable size. The impact of different instance and model properties on the difficulty of the problem is studied. The presence of economies of scales on several levels of the cost structure as well as partial facility closing are part of the main concerns. Further aspects include particular capacity constraints that involve integer rounding on the left hand side. It is shown how such capacity constraints can be useful in other applications, but increase the integrality gap of the problem. We derive valid inequalities to effectively reduce this integrality gap.

*Organization* This paper is organized as follows. Section 2 describes the relevant problem details. As the problem can be modeled as a facility location problem, the literature review in Sect. 3 focuses on relevant extensions in that domain. The mathematical formulation in Sect. 4 gradually extends the Capacitated Facility Location Problem to model the problem being addressed. This includes the particular capacity constraints, valid inequalities and additional features such as the relocation and partial closing of camps. Section 5 summarizes the results of the computational experiments performed. Two case studies in Sect. 6 illustrate the benefits of the proposed model when applied in practice. Finally, Sect. 7 concludes the work.

## 2 Problem description

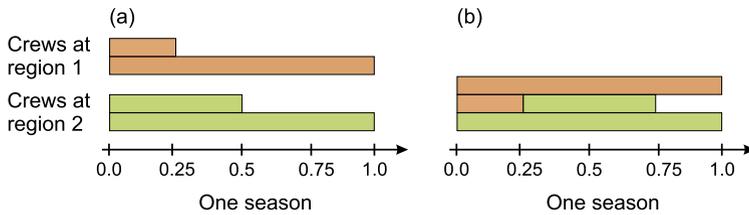
Based on an existing strategic plan, the logging company provides a harvesting plan for the next five years. Each year is divided into two seasons: winter and summer, each with a certain number of available working days. Depending on the geographical location, some regions will be logged more in winter whereas other regions will be logged more in summer. Each region is defined by its estimated log volume (measured in  $\text{m}^3$ ) that is subject to harvesting (it may be part of the strategic decision that not the entire region will be harvested) within each season and the length of the road (measured in km) that has to be constructed in that region in order to access the logging areas and transport the log.

### 2.1 Work crews, demands and hosting capacities

There are two types of work crews: logging and road construction. Crews of the same type contain the same number of members. The members of a crew always stay together during work and are hosted at the same accommodation. For each logging region and season, a logging and road construction demand is given. Based on given productivity rates for the work crews one can compute the average number of crews necessary to cover the demand at each region for each season.

*Example* Logging crews work 100 days within a given season and cut  $180 \text{ m}^3$  per day, i.e.,  $18,000 \text{ m}^3$  within the season. A certain region holds a total demand of  $27,000 \text{ m}^3$  for the season. Throughout 50 days, two logging crews will be working (i.e.,  $2 \cdot 50 \cdot 180 \text{ m}^3 = 18,000 \text{ m}^3$ ). The other 50 days, a single logging crew will be working (i.e.,  $1 \cdot 50 \cdot 180 \text{ m}^3 = 9,000 \text{ m}^3$ ). This results in an average allocation of  $27,000/18,000 = 1.5$  logging crews in that season.

As the operational assignment of logging crews is not our final concern, we can assume that the crews of each working type are flexible with respect to the days they work within each season. That is, if a crew works only a few days in a season, we may assume that



**Fig. 1** Example of logging demands hosted at the same accommodation

the exact days do not matter. In our example, it does not matter in which of the 100 days we use two crews and in which we use only one crew. In practice, a work crew may work a number of days in one region and then in another region in the same season. To determine the minimum capacity necessary to host all work crews allocated to a certain accommodation, consider the following example.

The workers from two regions are hosted at the same camp. One region has an average demand of 1.5 logging crews and 0.7 road construction crew. The other region has an average demand of 1.25 logging crews and 0.5 road construction crew. Figure 1(a) illustrates this scenario for the logging crews. In total, we have a demand of  $1.5 + 1.25 = 2.75$  logging crews and  $0.7 + 0.5 = 1.2$  road construction crews. Hence, for 75 % of the time during the season there will be  $\lceil 2.75 \rceil = 3$  logging crews and 25 % of the time there will be  $\lfloor 2.75 \rfloor = 2$  logging crews, which is illustrated in Fig. 1(b). In the same way, for 20 % of the season there will be  $\lceil 1.2 \rceil = 2$  road construction crews and for the other 80 % there will be only  $\lfloor 1.2 \rfloor = 1$  road construction crew. Assuming that a logging crew has six workers and a road construction crew has three workers, we will need accommodation for  $\lceil 2.75 \rceil \cdot 6 + \lfloor 1.2 \rfloor \cdot 3 = 18 + 6 = 24$  workers. To determine the minimum capacity of an accommodation, we can add the average numbers of crews allocated to this accommodation and round up the sum to the next highest integer (for each crew type).

**Transportation** Workers are usually transported by pick-ups, using a given road network. Costs are composed of the travel and working time of the workers as well as the vehicle costs, i.e., renting and gas. An additional salary has to be paid if a certain transportation time (usually one hour per day) is exceeded. This makes large travel distances very costly. Workers of the same crew are transported in one or more vehicles. Workers of different crews do not share the same vehicle.

**Supervisors** In addition to the work crews, there are fixed numbers of logging and road construction supervisors. Supervisors have to be considered for the accommodation capacities and their individual transportation costs. Although it is not clearly predictable how many days a supervisor will be at which region, one may assume that their presence in a region is proportional to the demand for work crews at that region. Hosting regions for supervisors are often limited to accommodations with administrative units.

## 2.2 Camps and trailers

Certain accommodations for the workers may already exist. These accommodations can either be hosting options in villages or cities (e.g., apartments, hotels or the employees' own homes) in reasonable distance of the logging regions, or camps that are usually located in the forest close to the logging regions. Accommodations vary in their capacity and their

hosting costs. Camps are composed of trailers. A trailer contains the infrastructure to host a certain number of workers. In practice, trailers of different capacities are available. However, for the purpose of this study, we may assume that the trailer with a capacity for twelve persons is the most common one and hence all trailers have the same capacity. In addition to the trailers that host workers, a camp contains a number of additional trailers that provide complementary, but necessary infrastructure, such as a kitchen and leisure facilities. The number of additional trailers directly depends on the total hosting capacity of the camp, i.e., the number of hosting trailers. In the following, we will measure the capacity of a camp by the number of hosting trailers. Hence, the construction costs for a number of hosting trailers already include the costs for the necessary number of additional trailers.

Trailers can be either open or closed. Only open trailers are available for use. Trailers that are not in use have to be closed, involving one-time closing costs. Once a trailer is closed, it cannot be used in subsequent seasons until it is reopened, involving one-time reopening costs. Closing or reopening operations can be performed before each season. Costs for such operations usually involve economies of scale in the number of hosting trailers, since common resources are shared. The use of hosting trailers to accommodate workers involves two types of daily costs: fixed costs for each open trailer (including the cost for the trailer itself, its equipment, the cook, etc.) and variable costs (food, etc.) for each worker. The fixed costs are paid for each open trailer per day. Costs for closed trailers are so small that they do not have to be considered. Variable costs are paid for each worker hosted at the camp. If a trailer is open, its fixed costs have to be paid throughout the entire season, independent of its use. All costs may follow the principle of economies of scale, i.e., the larger the quantity, the lower the price-per-worker/trailer. New camps can only be constructed at certain places from a given set of potential locations. It is very common that several logging regions are served by workers from the same accommodation. Though it is rare, one logging region may also be served by workers from different accommodations.

### 2.3 Capacity expansion and camp relocation

At certain points during the planning it may be interesting to increase the capacity of existing camps. Such capacity expansion is performed by adding new trailers. It is assumed that the cost of adding  $n$  trailers is the same as the construction of a new camp with  $n$  trailers. Trailers may also be permanently shut down. For the sake of simplicity, it is assumed that this is done by closing these trailers.

Logging regions are not equally harvested every year. That is, a camp may be close to logging regions with demands in certain years, but far away from logging regions that will be harvested afterwards. Instead of constructing a new camp, which involves high costs, camps can be moved from one location to another. The relocation of camps can only be performed once a year, before the summer season. The distance between the origin and destination for a relocation has very little impact on the total relocation costs. We may thus assume that the total cost for relocating a camp depends only on the camp size (i.e., the number of trailers it includes). All trailers have to be closed before relocation. After the relocation, all trailers that are supposed to be in use have to be reopened again. In theory, camps from two distinct locations can also be joined to further reduce the costs per unit. Trailers from the same camp could also be relocated to distinct locations. In practice, these features are observed rather rarely. For the sake of simplicity, it is hence assumed that camps can only be relocated as a whole and that two different camps cannot be merged at the same location.

## 2.4 Objective

Given that all logging and road construction demands must be covered, we must ensure that sufficient accommodations are available to host the workers. We want to minimize the total costs, which are composed of two parts:

- All costs involved in providing the necessary accommodations: camp construction, camp relocation, maintenance for open trailers, closing and reopening of trailers and hosting costs for workers.
- The transportation costs between the accommodations and the logging regions. This includes the costs for using the vehicles and an additional salary for long transportation times.

A solution to the problem consists of the following information, given for each of the seasons in each of the years of the planning horizon:

- For each camp construction: the location and camp size.
- For each camp relocation: the origin, destination and size of the relocated camp.
- For each camp: the number of trailers that will be closed or reopened.

An insight into the suggested assignment of work crew demands to the accommodations may also be interesting for decision-makers. The assignment is necessary to determine the minimum level of camp capacities. However, it is not explicitly part of the problem solution.

Throughout this work, we will refer to this problem as the *Camp Size and Location Problem (CSLP)*.

## 3 Literature review

The forestry sector has been an extensive user of Operations Research (OR) methods for strategic, tactical and operational planning. Optimization is mainly used for supply chain design (D'Amours et al. 2008), harvesting (Bredström et al. 2010) and transportation planning (Carlsson et al. 2009). Strong interest is shown by both the public and private sector, typically in countries where logs represent a large portion of the net exports, such as Canada, Chile, New Zealand and the Scandinavian countries. Several recent surveys provide broad overviews of optimization in the forestry sector (see, e.g., D'Amours et al. 2008; Rönnqvist 2003; Weintraub and Romero 2006).

Rönnqvist (2003) compares different planning levels in terms of planning horizon, allowable solution time and required solution quality. These characteristics strongly vary among the different applications. Board cutting is individually decided for each tree and has to be optimally solved within less than a second. Harvesting plans typically cover an entire year. Such forest management plans have to be evaluated quickly to allow manual comparisons. Thus, for problems of this category, near optimal solutions are desired within a few hours of computation time. However, the planning includes a strategic outlook for more than 100 years. To the best of our knowledge, the problem of locating logging camps has not yet been addressed in the OR literature. Its solution requirements are similar to those of road planning: one aims at near-optimal solutions, planning includes decisions for five years and one can allow computation times of several hours. Mathematical programming appears to be an appropriate tool, since it provides high quality solutions and it allows to model particular industrial constraints.

Several known problems present features similar to those found in the CSLP. Such problems typically belong to the family of Facility Location Problems. The CSLP can be formulated as an extension of the well studied Capacitated Facility Location Problem (CFLP), which aims at finding the optimal locations to construct an unknown number of facilities with capacity constraints. All customer demands have to be covered and the total costs, usually composed by costs for facility construction, production and transportation, are minimized. In the last decades, practical needs led to many extensions of the CFLP such as multiple periods, multiple commodities, multiple capacity levels and multiple stages. As demands are likely to change over time, many models focused on the dynamic (i.e., multi-period) case of the problem in order to address dynamic aspects such as capacity reduction, expansion and relocation.

The diversity, importance and maturity of facility location problems has been confirmed by many recent literature surveys (Hamacher and Nickel 1998; Klose and Drexl 2005; Melo et al. 2009; Reville and Eiselt 2005; Reville et al. 2008). Melo et al. (2009) focus on the context of supply chains. Smith et al. (2009) review the development of location analysis from its early beginning and highlights today's most important applications. Many of the extensions proposed for the CFLP can be found in the proposed CSLP. Camps are translated to facilities and hosting demands to customers. The relevant literature regarding these features will now be reviewed.

*Dynamic facility location problems* The CSLP contains strong dynamic aspects, since logging regions tend to be harvested within a few seasons. Hence, a customer may have high demands in some time periods and no demand at all in the other periods. Early works in the domain of dynamic facility location were initiated by Ballou (1968) and Wesolowsky (1973). Recent works include Albareda-Sambola et al. (2009), Canel et al. (2001), Dias (2006), Melo et al. (2005), Peeters and Antunes (2001), Shulman (1991) and Troncoso and Garrido (2005). Many more references can be found in the previously cited reviews as well as in the one of Owen and Daskin (1998), which focuses on approaches that are based on either dynamic or stochastic facility location problems.

In addition to the optimal timing and sizes for facility construction, further dynamic features have been found beneficial to adapt to changing demand and market conditions. Capacity expansion has been incorporated by Melo et al. (2005), Peeters and Antunes (2001) and Troncoso and Garrido (2005). Capacity reduction or facility shut-down is addressed by Canel et al. (2001), Dias (2006), Melo et al. (2005) and Peeters and Antunes (2001). In an early work, Wesolowsky and Truscott (1975) considered a simple case of relocation of facilities. Melo et al. (2005) provide an extensive modeling framework for dynamic multi-commodity facility location problems. Their model focuses on the relocation of existing facilities and gradual capacity transfer from existing facilities to new ones while considering generic multi-level supply chain network structures.

*Multiple commodities* In some applications, customers have demands for several distinct commodities. The models must then distinguish between the different commodities to satisfy the demand for each of them as well as to control their capacity at the facilities. In the context of the CSLP, the different work crew types (i.e., logging crews and road construction crews) and supervisors can be modeled as different commodities.

In the multi-commodity facility location literature, models commonly assume that the customers have an individual demand for each commodity. However, on the facility side, the capacity constraints can be formulated in two different ways:

1. Each facility holds an individual capacity for each of the commodities.
2. Each facility holds a global capacity for the sum of all commodities.

The first option is the more common one in the literature (Canel et al. 2001; Geoffrion and Graves 1974; Lee 1991; Warszawski 1973). In the CSLP, we rather consider the second case. While customers have a demand distinguished between the different commodities, the total capacity at the camps applies to the sum of all workers, whether they are logging or road construction workers. This idea of a common capacity for all commodities is also followed in the modeling framework of Melo et al. (2005).

*Multiple capacity levels* The presence of production capacities automatically raises the question of the dimension of such capacities. While some applications allow for several facilities at the same place, most consider only one facility per location. Facilities may have fixed capacities or may choose among different capacity levels. Often, facility construction and unit production costs follow the principle of economies of scale, i.e., the larger the facility, the cheaper the price per unit in terms of facility construction and commodity production. One finds this feature in the CSLP, where camps are composed of trailers. The more hosting trailers exist, the larger the capacity and the better common resources (such as supplementary infrastructure) are shared. The choice of different capacity levels allows to represent such economies of scale.

Early works considering different capacity levels are Lee (1991), Shulman (1991) and Sridharan (1991). The choice of the capacity level is modeled as an additional variable index, having only one variable of a certain capacity level active for each facility. The cost part in the objective function thus corresponds to a piecewise linear function. In the literature, this has been the most common way to represent such cost functions (Paquet et al. 2004; Troncoso and Garrido 2005).

Holmberg (1994) and Holmberg and Ling (1997) introduce an incremental approach to model staircase functions, where all variables up to the chosen capacity level are active. Similar approaches have since been adapted to more complex problems (Correia and Captivo 2003; Gouveia and Saldanha da Gama 2006).

*Conclusions* Many of the features found in the CSLP have already been addressed in isolation in the facility location literature. However, very few models consider modular capacity levels in a dynamic context (Melo et al. 2005; Peeters and Antunes 2001; Shulman 1991; Troncoso and Garrido 2005). These works do not address dynamic features such as facility closing/reopening or relocation. The closest related works are those of Melo et al. (2005) and Troncoso and Garrido (2005). The latter authors represent economies of scale for facility construction, but not for operational costs. Capacity relocation is also not considered. Melo et al. (2005) focus on capacity relocation, but consider modular capacity decisions only for relocation.

While many models consider closing an entire facility or reducing its capacity, none of the reviewed works present the possibility of partially or entirely deactivating a facility for a certain time period, as it is possible with trailers in logging camps. In addition, the capacity constraints found in the CSLP have not yet been addressed in the context of facility location problems.

#### 4 Mathematical formulation

The CSLP can be modeled as an extension of the CFLP. Some of the additional features have been considered in variations of that classical problem. However, to the best of our

knowledge, no extension of the CFLP considered all features at the same time. In particular, two of them have not been mentioned in the related literature:

1. Round-up (integer) capacity constraints for the camps.
2. Partial closing and reopening of trailers throughout the planning periods.

In the following, we will model the CSLP by extending the CFLP in two steps. In a first step, a formulation for a dynamic modular (i.e., multiple capacity levels) multi-commodity Facility Location Problem with multi-source assignment is studied. This problem will be referred to as the DMCFLP. Then, the dynamic features are added, namely the relocation of camps and the closing and reopening of trailers. This problem represents the CSLP as described above.

The intermediate problem, namely the DMCFLP, is explored mainly due to two reasons. First, to explore the impact of the additional features on the solution difficulty. Second, all DMCFLP solutions are essentially feasible for the CSLP. As we will see later on, DMCFLP solutions of good quality can be obtained much easier than solutions for the CSLP. Using DMCFLP solutions as starting solutions can be helpful to solve the complete CSLP.

#### 4.1 The DMCFLP—an extension of the CFLP

The classical CFLP, as presented by Sridharan (1995), is extended. To be more precise, the following features are added:

- Multiple periods. We study the problem in a dynamic context, i.e., over multiple time periods with independent demands.
- Multiple commodities. We assume the existence of different commodities, one for each work crew type. Each customer may have independent demands for each of these commodities.
- Multiple capacity levels. We assume that a facility may have different capacities, i.e., different numbers of hosting trailers. These capacities are modular and can represent cost structures involving economies of scale.

Due to its additional characteristics, we refer to this problem as the *Dynamic Modular Multi-Commodity Facility Location Problem (DMCFLP)*.

##### 4.1.1 Input data and decision variables

*Input data* Consider the following input data:

- $I$ —set of potential camp locations (facilities).
- $J$ —set of logging/road construction regions (customers).
- $K$ —set of possible camp sizes (with respect to the number of hosting trailers),  $K = \{1, 2, \dots, \bar{K}\}$ .
- $P$ —set of existing work crew types (commodities).
- $T$ —set of seasons (time periods),  $T = \{1, 2, 3, \dots, |T|\}$ .
- $N_p$ —number of workers in a crew of type  $p$ .
- $d_{jpt}$ —demand (in number of crews) for commodity  $p \in P$  in region  $j \in J$  and period  $t \in T$ .
- $u_{ik}$ —total capacity (in number of workers) of a camp of size  $k \in K$  at location  $i \in I$ .
- $c_{ik}^C$ —construction cost of a camp of size  $k \in K$  at location  $i \in I$ .

- $c_{ijkpt}^V$ —variable operational costs (including transportation and hosting costs) for the entire time period  $t \in T$  for one crew of working type  $p \in P$  accommodated at a camp of size  $k \in K$  at location  $i \in I$  and working at region  $j \in J$ . The total cost is typically not linear with respect to the Euclidean distance between the work region and the accommodation.

*Decision variables* The decision variables are:

- $x_{ijkpt} \in \mathbb{R}^+$ —total demand (in number of crews) of crew type  $p \in P$  assigned from a camp of size  $k \in K$  at location  $i \in I$  to region  $j \in J$  at time period  $t \in T$ .
- $y_{ik} \in \{0, 1\}$ —1, if a camp of size  $k \in K$  is constructed at location  $i \in I$  at the beginning of the horizon, 0 otherwise.

#### 4.1.2 Mathematical model

The model is given by:

$$\min \sum_{i \in I} \sum_{k \in K} c_{ik}^C y_{ik} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{p \in P} \sum_{t \in T} c_{ijkpt}^V x_{ijkpt} \tag{1}$$

$$\text{s.t. } \sum_{i \in I} \sum_{k \in K} x_{ijkpt} = d_{jpt}, \quad \forall j \in J; \forall p \in P; \forall t \in T \tag{2}$$

$$\sum_{p \in P} \sum_{j \in J} N_p x_{ijkpt} \leq u_{ik} y_{ik}; \quad \forall i \in I; \forall k \in K; \forall t \in T \tag{3}$$

$$\sum_{k \in K} y_{ik} \leq 1; \quad \forall i \in I \tag{4}$$

$$x_{ijkpt} \leq d_{jpt} y_{ik}; \quad \forall i \in I; \forall j \in J; \forall k \in K; \forall p \in P; \forall t \in T \tag{5}$$

$$x_{ijkpt} \in \mathbb{R}^+; \quad \forall i \in I; \forall j \in J; \forall k \in K; \forall p \in P; \forall t \in T \tag{6}$$

$$y_{ik} \in \{0, 1\}; \quad \forall i \in I; \forall k \in K \tag{7}$$

The objective function (1) minimizes the camp construction cost and the operational costs. Note that the operational costs  $c_{ijkpt}^V$  are composed by both transportation and hosting costs. The transportation costs depend on the distance between both locations  $i$  and  $j$  as well as the type of crew  $p$ . The hosting costs depend on the camp size  $k$  as well as the crew type  $p$ .

The set of constraints (2) guarantees that all customer demands are satisfied. Note that demands are likely to be fractional, as illustrated in Fig. 1. Constraints (3) require that the hosting demands assigned to each camp do not exceed the camp capacities. Constraints (4) ensure that only one capacity level is selected for each facility. The set of valid inequalities (5), also referred to as *Strong Inequalities (SI)* (Gendron and Crainic 1994), provide a stronger upper bound for the demand assignment variables. Computational experiments show that CPLEX solves the problem more effectively when adding only the violated SIs (using CPLEX user cuts) than when adding all SIs *a priori* or not adding them at all.

*Non-movable accommodations* In addition to logging camps, we may model accommodations such as motels and apartments to host workers. We do so by representing them as a restricted case of a camp, with two types of information: hosting costs and total capacity. Such accommodations possess a single capacity level and cannot be relocated.

### 4.2 Round-up capacity constraints

As explained above, the CSLP involves particular capacity constraints where the sum of all demands assigned to a certain accommodation is rounded up to the next integer value. Adding, for example, demands of 1.5 crews and 1.25 crews, one only needs a total capacity for three crews (if all crews are hosted at the same camp) instead of four (compare Fig. 1).

We introduce additional integer variables  $z_{ikpt}$  for the integer rounding, indicating the total number of crews of type  $p$  assigned to a size  $k$  camp at location  $i \in I$  at period  $t \in T$ . The existing capacity constraints (3) are replaced by two new constraints (8) and (9), which we will refer to as the *round-up capacity constraints (RUC)*. Instead of using the continuous sum of the facility/customer assignment variables ( $x$  variables), the capacity constraints (9) take into account the next highest integer value, bounded by the  $z$  variables in constraints (8):

$$\sum_{j \in J} x_{ijkpt} \leq z_{ikpt}; \quad \forall i \in I; \forall k \in K; \forall p \in P; \forall t \in T \tag{8}$$

$$\sum_{p \in P} N_p z_{ikpt} \leq u_{ik} y_{ik}; \quad \forall i \in I; k \in K; \forall t \in T \tag{9}$$

$$z_{ikpt} \in \mathbb{Z}^+; \quad \forall i \in I; \forall k \in K; \forall p \in P; \forall t \in T \tag{10}$$

This type of capacity constraints is likely to appear in other applications. In the context of facility location problems, scenarios can be modeled where a facility may not be able to produce any arbitrary amount of a product, but only modular sized packages of products.

#### 4.2.1 Strengthening the formulation

Experiments have shown that the average integrality gap increases significantly (see Sect. 5.2 for details) when using round-up capacity constraints (8)–(10) instead of the usual constraints (3). Consider the following *aggregated demand* inequalities which are known to be redundant for the linear relaxation of the model:

$$\sum_{i \in I} \sum_{k \in K} u_{ik} y_{ik} \geq \sum_{p \in P} \sum_{j \in J} d_{jpt} N_p; \quad \forall t \in T$$

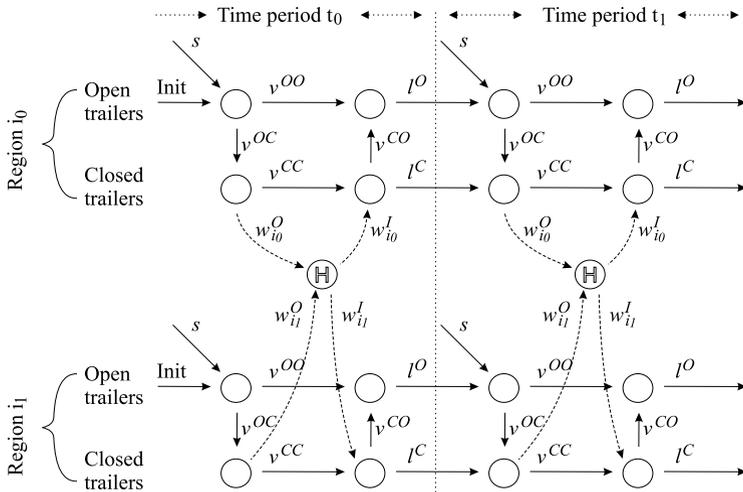
We will now strengthen these inequalities, based on the fact that  $z$  is integer. Substituting (2) in (8) shows that one can always round up the sum of all demands from different regions for the same product. We replace the right hand side (RHS) of the previous inequality by  $D_t$ , where:

$$D_t = \sum_{p \in P} \left\lceil \sum_{j \in J} d_{jpt} \right\rceil N_p; \quad \forall t \in T$$

We now express the resulting inequality in terms of the number of trailers instead of the number of crews. Assuming that each trailer hosts exactly  $M$  workers, i.e.,  $u_{ik} = Mk$ , we have:

$$\sum_{i \in I} \sum_{k \in K} k y_{ik} \geq \frac{D_t}{M}; \quad \forall t \in T$$

These inequalities state the minimum number of open trailers necessary to satisfy all customer demands. We know that the RHS, the minimum number of open trailers, is always integer. We can thus replace the RHS by  $S_t$ , where:



**Fig. 2** Network model to manage open and closed trailers at each location

$$S_t = \left\lceil \frac{D_t}{M} \right\rceil; \quad \forall t \in T$$

In a final step, we aim at reducing the coefficients of the  $y$  variables on the left hand side. Suppose that  $\bar{K} > S_t$ . It is then sufficient that only one  $y_{ik'}$  with  $k' \geq S_t$  is active in order to satisfy the entire customer demand in the integer solution. That is, we may set the coefficient of a variable  $y_{ik'}$  to  $S_t$  whenever  $k' \geq S_t$ :

$$\sum_{i \in I} \sum_{k \in K} \min\{k, S_t\} y_{ik} \geq S_t; \quad \forall t \in T \tag{11}$$

In the following, we will refer to these constraints as the *strengthened aggregated demand (SAD)* inequalities.

### 4.3 The CSLP—adding partial camp closing, relocation and modular costs

In this section, the previous model will be extended with the following features that may appear in a dynamic context:

1. Construction of new camps/trailers at any time period.
2. Closing and reopening of trailers at any time period.
3. Relocation of camps at any time period.
4. Modular costs for trailer closing/reopening and camp relocation.

This problem corresponds to the CSLP. A network flow structure, illustrated in Fig. 2, is added on top of the previously introduced model to manage the first three features. For each time period, two nodes for open trailers and two nodes for closed trailers are used. Arcs between these nodes represent certain operations to modify the number of open and closed trailers at each location and to relocate them to other locations. The flow on these arcs indicates the number of trailers involved in the corresponding operation. New trailers can be constructed at the beginning of any season ( $s$  arcs). Open trailers can be closed ( $v^{OC}$  arcs) and closed trailers can be reopened ( $v^{CO}$  arcs). The arcs  $v^{OO}$  represent trailers that

were open at the beginning of the season and remain open during the current season. The arcs  $v^{CC}$  indicate closed trailers that are not relocated to another region. These trailers can still be reopened for the current season. Finally,  $l^O$  and  $l^C$  indicate the number of trailers that are open and closed, respectively, at each location throughout the entire season.

Relocation is allowed only for closed trailers. One could model relocation by the use of direct arcs between all location pairs. However, this would result in very large models. Experiments showed that this significantly increases the model size and therefore also the difficulty of solving the problem. Instead, relocation is modeled by the use of a central node, here referred to as a *hub node* ( $H$ ). The flow of relocated trailers is first passed to the hub node ( $w^O$  arcs) and then further distributed to another location ( $w^I$  arcs).

### 4.3.1 Input data and decision variables

*Additional input data* In addition to the previously introduced input data, additional parameters are considered. These data may already consider economies of scale with respect to  $k$ , the number of trailers involved in the operation:  $c_k^{TO}$  and  $c_k^{TC}$  are the costs to reopen and close  $k$  trailers of the same camp, respectively. The maintenance costs for a camp with  $k$  open trailers during season  $t$  is given by  $c_{kt}^M$ . Finally,  $c_k^R$  represents the costs for relocating a camp with  $k$  closed trailers.

*Additional decision variables* To incorporate the new features, some variables have to be extended and new variables have to be added to the model. Binary variables  $y_{ikt}$  now indicate whether the camp located at  $i$  has  $k$  open hosting trailers during period  $t$ . A separate binary variable  $s_{iqt}$  indicates the construction of  $q$  new trailers at location  $i$  before period  $t$ . In addition, arc flow variables for the network are added to manage the closing and reopening of trailers:  $l_{it}^O, l_{it}^C, v_{it}^{OO}, v_{it}^{OC}, v_{it}^{CO}, v_{it}^{CC}, w_{it}^O$  and  $w_{it}^I$ .

Finally, binary variables are needed to incorporate modular costs:  $v_{ikt}^{BCO}$  and  $v_{ikt}^{BOC}$  indicate whether  $k$  trailers are reopened or closed, respectively, at location  $i$  before time period  $t$ . Variables  $w_{ikt}^{BO}$  and  $w_{ikt}^{BI}$  indicate whether a size  $k$  camp is relocated from or to, respectively, location  $i$  before period  $t$ . The relocation of a camp of size  $k'$  from location  $i_1$  to location  $i_2$  at time period  $t'$  is thus performed by selecting the two variables  $w_{i_1k't'}^{BO}$  and  $w_{i_2k't'}^{BI}$ .

### 4.3.2 Mathematical model

*Objective function* The objective function minimizes all costs: maintenance for open trailers, operational hosting and transportation, trailer construction, camp relocation and trailer reopening and closing. Note that each camp relocation involves two binary variables  $w_{ikt}^{BO}$  and  $w_{ikt}^{BI}$ , while only one of them has to be considered in the objective function to attribute the relocation costs:

$$\begin{aligned}
 \min \quad & \sum_{i \in I} \sum_{k \in K} \sum_{t \in T} c_{kt}^M y_{ikt} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{p \in P} \sum_{t \in T} c_{ijkpt}^V x_{ijkpt} \\
 & + \sum_{i \in I} \sum_{q \in K} \sum_{t \in T} c_{in}^C s_{iqt} + \sum_{i \in I} \sum_{k \in K} \sum_{t \in T} c_k^R w_{ikt}^{BO} \\
 & + \sum_{i \in I} \sum_{k \in K} \sum_{t \in T} c_k^{TO} v_{ikt}^{BCO} + \sum_{i \in I} \sum_{k \in K} \sum_{t \in T} c_k^{TC} v_{ikt}^{BOC} \tag{12}
 \end{aligned}$$

*Demand and capacity constraints* The constraints representing the part of the facility location problem are identical to the ones in the previously presented model. However, the  $y$  variables now represent the number of open trailers at each location and time period:

$$\sum_{i \in I} \sum_{k \in K} x_{ijkpt} = d_{jpt}; \quad \forall j \in J; \forall p \in P; \forall t \in T \tag{13}$$

$$\sum_{j \in J} x_{ijkpt} \leq z_{ikpt}; \quad \forall i \in I; \forall k \in K; \forall p \in P; \forall t \in T \tag{14}$$

$$\sum_{p \in P} N_p z_{ikpt} \leq u_{ik} y_{ikt}; \quad \forall i \in I; \forall k \in K; \forall t \in T \tag{15}$$

$$\sum_{k \in K} y_{ikt} \leq 1; \quad \forall i \in I; \forall t \in T \tag{16}$$

$$x_{ijkpt} \leq d_{jpt} y_{ikt}; \quad \forall i \in I; \forall j \in J; \forall k \in K; \forall p \in P; \forall t \in T \tag{17}$$

*Flow conservation and consistency constraints* The network is modeled by the following constraints. Constraints (18), (19), (20) and (21) represent the first nodes for open and closed trailers and the second nodes for open and closed trailers, respectively. Note that the variables  $l_{it}^O$  and  $l_{it}^C$  do not exist for  $t = 0$ , i.e., in constraints (18) and (19), we have  $l_{i(t=0)}^O = 0$  and  $l_{i(t=0)}^C = 0$ . If a region  $i \in I$  already possesses a camp at the beginning of the planning horizon, then a constant  $\Gamma_{it} > 0$  (with  $t = 1$ ) indicates the number of hosting trailers of that camp. Clearly,  $\Gamma_{it} = 0$  for all  $t > 1$ . Constraints (22) guarantee that the number of existing trailers at a camp never exceeds the maximum camp size, while (23) link the  $y$  variables to the number of open trailers:

$$\Gamma_{it} + l_{i(t-1)}^O + \sum_{q \in K} q s_{iqt} = v_{it}^{OO} + v_{it}^{OC}; \quad \forall i \in I; \forall t \in T \tag{18}$$

$$l_{i(t-1)}^C + v_{it}^{OC} = v_{it}^{CC} + w_{it}^O; \quad \forall i \in I; \forall t \in T \tag{19}$$

$$v_{it}^{OO} + v_{it}^{CO} = l_{it}^O; \quad \forall i \in I; \forall t \in T \tag{20}$$

$$v_{it}^{CC} + w_{it}^I = v_{it}^{CO} + l_{it}^C; \quad \forall i \in I; \forall t \in T \tag{21}$$

$$l_{it}^O + l_{it}^C \leq \bar{K}; \quad \forall i \in I; \forall t \in T \tag{22}$$

$$\sum_{k \in K} k y_{ikt} = l_{it}^O; \quad \forall i \in I; \forall t \in T \tag{23}$$

*Relocation consistency constraints* Equalities (26) enforce that if a camp of size  $k$  is removed from a location, then a camp of the same size must be placed at another region. They ensure that trailers of different camps will not be mixed if they are relocated at the same time period. Constraints (24) ensure that camps are only relocated as a whole, i.e., no trailers remain at the location if a camp is relocated. Constraints (25) say that a camp can only be relocated to locations where no other camps exist. Constraints (26) ensure that camps from different locations are not merged. Although redundant, constraints  $\sum_{k \in K} w_{ikt}^{BO} \leq 1$  and  $\sum_{k \in K} w_{ikt}^{BI} \leq 1$  are explicitly added to the model, since they help CPLEX generate further cuts.

$$v_{it}^{CC} + v_{it}^{OO} \leq \bar{K} \left( 1 - \sum_{k \in K} w_{ikt}^{BO} \right); \quad \forall i \in I; \forall t \in T \tag{24}$$

$$v_{it}^{CC} + v_{it}^{OO} - \sum_{q \in K} q s_{iqt} \leq \bar{K} \left( 1 - \sum_{k \in K} w_{ikt}^{BI} \right); \quad \forall i \in I; \forall t \in T \tag{25}$$

$$\sum_{i \in I} w_{ikt}^{BO} = \sum_{i \in I} w_{ikt}^{BI}; \quad \forall k \in K; \forall t \in T \tag{26}$$

*Linking constraints for modular costs* Linking constraints as suggested by Melo et al. (2005) are used to link the continuous arc flow variables to the binary variables for modular decisions:

$$\sum_{k \in K} kv_{ikt}^{BCO} = v_{it}^{CO}; \quad \forall i \in I; \forall t \in T \tag{27}$$

$$\sum_{k \in K} kv_{ikt}^{BOC} = v_{it}^{OC}; \quad \forall i \in I; \forall t \in T \tag{28}$$

$$\sum_{k \in K} kw_{ikt}^{BO} = w_{it}^O; \quad \forall i \in I; \forall t \in T \tag{29}$$

$$\sum_{k \in K} kw_{ikt}^{BI} = w_{it}^I; \quad \forall i \in I; \forall t \in T \tag{30}$$

*Variable domains* Once the  $y$  variables are fixed, the remaining subproblem defined by the network flow structure can be stated as a *Minimum Cost Network Flow Problem*. All  $l^O$  arcs are then fixed according to the  $y$  values due to the equality constraints (23). Thus, the remaining network matrix has the *unimodularity property*. We could thus state all arc variables as continuous without losing their integrality property in the solution. However, we keep integrality on the arc variables, since experiments showed that it slightly facilitates the solution by CPLEX.

$$x_{ijkpt} \in \mathbb{R}^+; \quad \forall i \in I; \forall j \in J; \forall k \in K; \forall p \in P; \forall t \in T \tag{31}$$

$$z_{ikpt} \in \mathbb{Z}^+; \quad \forall i \in I; \forall k \in K; \forall p \in P; \forall t \in T \tag{32}$$

$$y_{ikt} \in \{0, 1\}; \quad \forall i \in I; \forall k \in K; \forall t \in T \tag{33}$$

$$s_{iqt} \in \{0, 1\}; \quad \forall i \in I; \forall q \in K; \forall t \in T \tag{34}$$

$$l_{it}^O, l_{it}^C, v_{it}^{CC}, v_{it}^{CO}, v_{it}^{OO}, v_{it}^{OC}, w_{it}^O, w_{it}^I \in \mathbb{Z}^+; \quad \forall i \in I; \forall t \in T \tag{35}$$

$$v_{ikt}^{BCO}, v_{ikt}^{BOC}, w_{ikt}^{BO}, w_{ikt}^{BI} \in \{0, 1\}; \quad \forall i \in I; \forall k \in K; \forall t \in T \tag{36}$$

Note that, for the CSLP, the SAD inequalities given by (11) are modified, replacing each variable  $y_{ik}$  by a variable  $y_{ikt}$ .

## 5 Computational experiments

### 5.1 Instance generation and experimentation environment

In order to test the robustness of the model, instances have been generated with different parameters. Certain data have been adapted from a real-world (RW) instance, based on data provided by a Canadian logging company (see Sect. 6.2). Key parameters are found to be the ones that may change the difficulty of the problem, namely:

- **Problem dimension.** Instances have been generated with the following dimensions (*#facility locations/#customers*): (10/20), (10/50), (50/50) and (50/100).

- **Distances and transportation costs.** For each of the problem sizes, three different networks have been randomly generated on squares of the following sizes: 300 km × 300 km, 380 km × 380 km and 450 km × 450 km. Transportation costs have been computed as explained in Sect. 2.1.
- **Number of commodities.** Demands are generated either only for logging and road construction (i.e., two commodities) or additionally for the corresponding supervisors (i.e., four commodities).
- **Concavity of the cost curves.** Two extreme cases are considered: construction and operational costs are either linear or concave. In addition, the cost curves given in the RW instance with linear construction costs and concave operational costs are considered.
- **Demand distribution.** The demand for each region within each season is randomly generated so that the total demand in each season throughout all regions is similar. For each region, the demand is either uniformly distributed over all seasons or randomly distributed over up to four seasons.
- **Cost distribution.** The ratio between camp construction/relocation and transportation costs is generated for different ratios. The transportation costs were set to 20 %, 100 % and 200 % of the original transportation costs indicated in the RW instance.
- **Initial demand coverage.** Instances are generated with different numbers of initially existing camps. The total capacity of such camps covers either 0 %, 50 % or 100 % of the total demand.

All generated instances contain ten time periods. Camp relocation costs and the costs to close or reopen trailers have been adapted from the RW instance. The maximum camp size  $\bar{K}$  has been chosen so that a single camp with  $\bar{K}$  trailers is capable to host the entire worker demand. The combination of all different configurations explained above resulted in 1296 instances. Experiments on all instances showed that instances are significantly easier to solve when the cost curves are linear or only two commodities (i.e., no demands for supervisors) are used. On the other hand, instances with 50 or more potential facility locations could virtually not be solved within the imposed time limit of one hour of computation time. The results presented throughout this paper are thus based on a subset of the instances described above. This subset includes 216 instances: all instances of reasonable size, i.e., (10/20) and (10/50), excluding those which are known to be easily solved, i.e., having only two commodities or linear cost curves.

The code has been written in C/C++ using the Callable Library of IBM ILOG CPLEX 12.3 and has been compiled and executed on openSUSE 11.3. Each problem instance has been run on a single AMD Opteron 250 processor (2.4 GHz), limited to 4 GB of RAM. If not stated otherwise, CPLEX computation times have been limited to 60 minutes.

## 5.2 Computational results

The following variants of the problem have been considered to investigate the impact of the different problem features on the difficulty of solving the problem:

- The DMCFLP as described in Sect. 4.1. Both versions without and with RUC constraints (round-up capacity constraints, see Sect. 4.2) and SAD inequalities (strengthened aggregated demand inequalities, see Sect. 4.2.1) are considered.
- The CSLP, as described in Sect. 4.3.

The SI valid inequalities, given by (5) and (17) for the DMCFLP and the CSLP, respectively, are very effective to strengthen the model. The integrality gap of the DMCFLP with

**Table 1** Comparing the solution quality for the DMCFLP without/with RUC constraints as well as without/with SAD inequalities after one hour of computation time

Inst size	# Inst	w/o RUC			w/ RUC w/o SAD			w/ RUC w/ SAD		
		gap %		#	gap %		#	gap %		#
		avg	max	ns	avg	max	ns	avg	max	ns
10/20	108	0.00	0.01	0	7.73	41.05	0	0.39	26.01	0
10/50	108	8.60	38.26	0	25.06	59.88	10	17.82	57.83	10
All	216	4.30	38.26	0	16.02	59.88	10	8.68	57.83	10

RUC constraints and SAD inequalities was found to be 20.3 % (average over the 216 selected instances). Adding the SI inequalities (5) to the model decreased the integrality gap to an average of 2.2 %. In CPLEX, valid inequalities can be added to the model either all *a priori* or dynamically (called *user cuts*), only those that are violated during the solution of the linear relaxation. In the following experiments, the SIs have been added as user cuts in the case of the DMCFLP. For the CSLP, all SIs have been added to the model *a priori*. Further experiments indicate that CPLEX performs best when the parameter *MIPEmphasis* is set to *feasibility*.

### 5.2.1 Impact of the RUC constraints and SAD inequalities

Computational experiments for the DMCFLP (performed on all 1296 instance described above) showed that the average integrality gap increased from 2.8 % to 6.0 % when the RUC (round-up capacity) constraints are used within the model. This indicates that the RUC constraints significantly complicate the solution of the problem. However, the additional use of the SAD inequalities reduces the average integrality gap to 1.4 %.

Table 1 summarizes the average optimality gaps after one hour of computation time. We compare three different versions for the DMCFLP. The version *w/o RUC* indicates the DMCFLP, defined by (1)–(7), with common capacity constraints (i.e., no round-up capacity constraints). The second version, denoted by *w/ RUC w/o SAD*, explores the impact of the round-up capacity constraints. This problem version is thus defined by (1), (2) and (4)–(10). Finally, we investigate the impact of the SAD inequalities. The version, denoted by *w/ RUC w/ SAD*, is thus defined by (1), (2) and (4)–(11). As previously mentioned, all SIs, given by inequalities (5) are added as CPLEX user cuts.

For each of the three versions we report average and maximum optimality gaps. The column *# ns* indicates the number of instances where either no feasible integer solution has been found or the solver ran out of memory. The results indicate that adding the round-up capacity constraints significantly complicates the solution of the problem. For ten instances, no feasible solution could be found. However, the additional use of the SAD inequalities proved quite effective to improve the optimality gap.

### 5.2.2 Solving the CSLP and solution properties

We now explore how the difficulty of solving the CSLP is affected by the RUC constraints and SAD inequalities. We also investigate the impact of different instance characteristics. We show relations between the optimal solutions of the DMCFLP and the CSLP by comparing the number of constructed and relocated trailers. This leads to the idea of using DMCFLP solutions as starting solutions for the CSLP. The impact of certain properties such

**Table 2** Comparing the solution quality after one hour of computation time using different solution approaches

Inst size	# Inst	CSLP w/o SAD			CSLP w/ SAD			CSLPHeur w/ SAD		
		gap %		# ns	gap %		# ns	gap %		# ns
		avg	max		avg	max		avg	max	
10/20	108	28.71	55.32	41	9.13	54.06	22	6.42	20.24	0
10/50	108	–	–	108	3.53	22.57	91	18.24	49.95	31
All	216	28.71	55.32	149	8.21	54.06	113	11.89	49.95	13

as the demand distribution over time, the initial camp capacity and the dimension of transportation costs is evaluated.

Table 2 compares the results for different solution approaches: two approaches based on conventional CPLEX optimization and a third approach which is explained further below. The first approach, denoted by *CSLP w/o SAD*, involves the solution of the CSLP defined by (12)–(36) using CPLEX. The second approach, denoted by *CSLP w/ SAD* additionally uses the SAD inequalities (11).

The table presents average and maximum optimality gaps when compared with the best known lower bound for each instance. In addition, the number of instances where no feasible integer solution has been found or the solver ran out of memory (*#ns*) is reported. As the results indicate, the SAD inequalities improve the performance of CPLEX. Feasible solutions can be found for 36 further instances and the solution quality improves significantly.

*DMCFLP warm start solutions for the CSLP* As we observed in the previous section, DMCFLP solutions of fair quality can easily be obtained. For the CSLP, we may have trouble to find any feasible integer solution at all. However, a feasible solution for the DMCFLP is also feasible for the CSLP. To convert an optimal DMCFLP solution into a feasible CSLP solution, the  $y$  variable values of the DMCFLP solution are fixed. CPLEX then heuristically finds feasible values for the missing variables (parameter *effortLevel* has been set to 3). Table 3 shows the average optimality gaps of the optimal DMCFLP solutions in the CSLP. The average optimality gap of such solutions (except for five instances of size (10/50) where no optimal DMCFLP solution has been found) is around 15 %. The results are then separated by instances with certain characteristics, namely the demand distribution along time as well as the initial demand coverage by existing camps. One would assume that DMCFLP solutions perform better for instances where the demand is uniformly distributed over time, since the relocation of camps seems less probable. However, the results do not show any clear evidence of a better performance.

On the other hand, the total capacity of existing camps seems to have more impact on the DMCFLP solution quality in the CSLP. The less camps initially exist, the better the DMCFLP solution quality. This is because in both versions camps have to be constructed. This is summarized in Table 4, which reports the average number of constructed and relocated trailers according to the demand distribution and the number of initially existing camps (only solutions with a proven optimality gap smaller than or equal to 10 % have been considered). Instances with demand uniformly distributed over all time periods tend to have less constructions and relocations than instances in which demand is irregularly distributed over time. In addition, the less camp capacity is initially available, the smaller the chance that existing camps are relocated instead of constructing new ones. Thus, new optimal placed camps in a DMCFLP solution are more likely to be a good choice for the CSLP as well.

**Table 3** The average optimality gaps of optimal DMCFLP solutions in the CSLP

Inst size	All	Demand distribution		Initial demand coverage		
		Uniform	Clustered	0 %	50 %	100 %
10/20	12.8	11.6	13.9	9.2	12.9	16.1
10/50	17.2	18.8	15.7	12.1	15.8	23.9
Total	14.9	15.0	14.8	10.6	14.4	19.8

**Table 4** The average number of constructed and relocated trailers within near optimal CSLP solutions

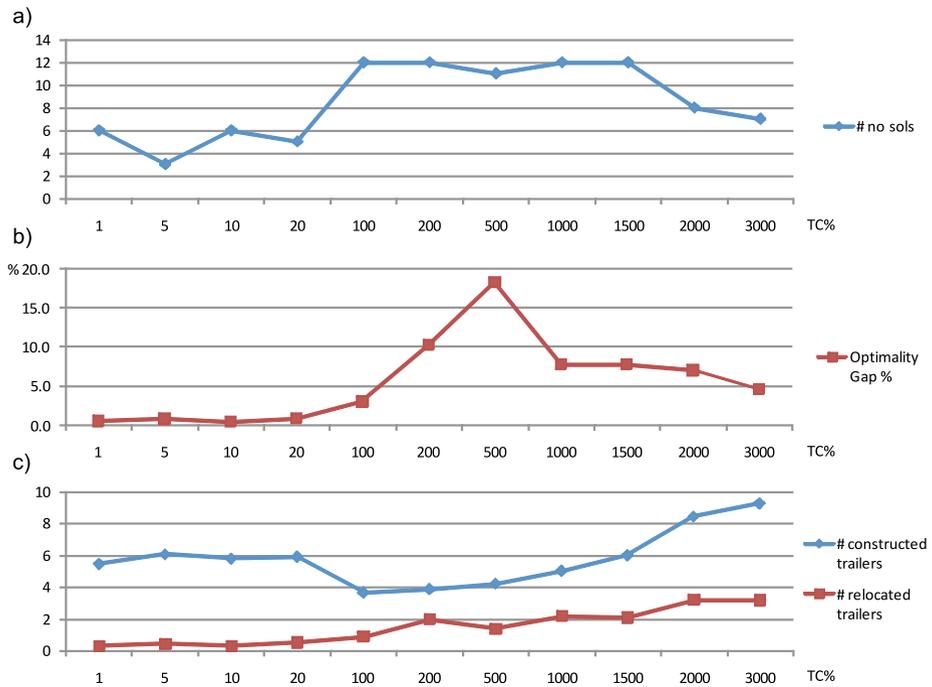
Inst size	Demand distribution		Initial demand coverage		
	Uniform	Clustered	0 %	50 %	100 %
# Constructions	4.9	6.7	7.8	4.6	2.3
# Relocations	0.8	1.1	0.0	1.2	1.8

We may thus use DMCFLP solutions as warm start solutions for the CSLP. The last three columns, denoted by *CSLPHeur w/ SAD*, in Table 2 indicate the results after one hour of computation time for the CSLP (w/ SAD), when the best DMCFLP (w/ RUC w/ SAD) solution obtained after one hour of computation time is used as a warm start solution. Compared with the conventional execution of the CSLP, CPLEX now finds feasible solutions for most of the instances while maintaining a similar average optimality gap.

*The impact of the cost ratio* The ratio between transportation costs and the costs to construct or relocate camps has also been found to have a strong impact on the difficulty of solving the problem. A total of 264 additional instances of the sizes (10/20) and (10/50) have been generated with eleven different transportation costs, set between 1 % and 3000 % of the original transportation costs given in the RW instance. We refer to this percentage as *TC %*. All instances contain sufficient camp capacities to cover 50 % of the average demand per season.

Figure 3(a) and (b) illustrate the difficulty of solving the generated instances for the CSLP subject to their *TC %* ratios (in one hour of computation time). For each of the *TC %* cost ratios, the number of instances where no feasible solution has been found (see Fig. 3(a)) and the average optimality gap of the final solutions (see Fig. 3(b)) are reported. The results indicate that the problem gets more difficult to solve when *TC %* = 100. With *TC %* values greater than 1500, it seems that the solution of the problem gets slightly easier again. Figure 3(c) shows the average number of constructed and relocated trailers within the final solutions (again, only solutions with a proven optimality gap smaller than or equal to 10 % have been considered). The results indicate that the number of constructed trailers grows faster than the number of relocations when the transportation costs increase.

*Yearly camp relocation* All previous experiments have assumed that camp relocation is allowed after each season. In the case of the Canadian logging company that provided the real-world instance, relocation is possible only once a year. We investigate the difficulty of solving this slightly simplified problem, considering all instances (*ISall*). We use the *CSLPHeur* approach, i.e., we first solve the DMCFLP with a time limit of one hour and then use the best solution as a starting solution for the CSLP, also limited to one hour of



**Fig. 3** The impact of the transportation cost ratio on (a) the number of CSLP instances where no solutions have been found, (b) the average optimality gaps and (c) the average number of constructed and relocated trailers in near optimal solutions

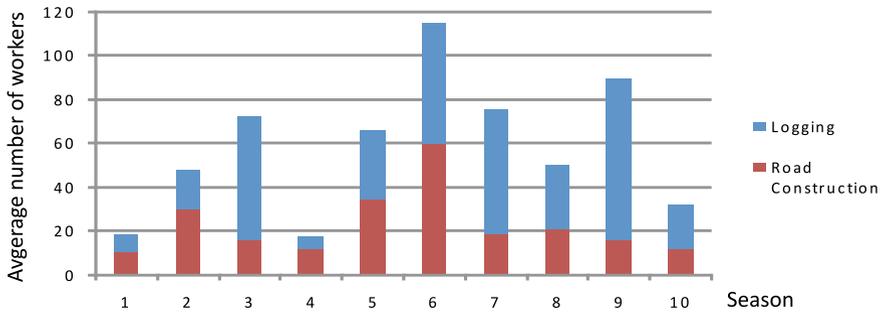
**Table 5** Results (*ISall*) with CSLPHeur when camp relocation is allowed only once a year

Inst size	# Inst	gap %	# ns	# opt	Time (s)
10/20	324	4.3	0	134	3992
10/50	324	14.6	17	24	5664
50/50	324	24.2	134	12	7173
50/100	324	19.7	295	31	7447
Total avg	1296	11.5	446	201	4984

computation time. The results, summarized in Table 5, show that instances of reasonable size (i.e., 10/20 and 10/50) can be fairly well solved. Most of the larger instances exceed either the given memory limit of 4 GB or CPLEX capabilities to solve the problem in the given time limit.

### 6 Case study

In this section, we analyze the planning solutions proposed by our model for two planning periods of our industrial partner. Each of the two planning periods spans five years. Each year is divided into a summer and a winter season. For the first planning period, we consider



**Fig. 4** Total demand (in average number of workers per day) throughout all seasons

the activities performed by the company throughout the harvest period 2006 to 2010. We aim at simulating the decisions made by the company and compare them with the decisions suggested by the mathematical model. The second planning horizon considers the harvest planning for the next five years, starting in 2011.

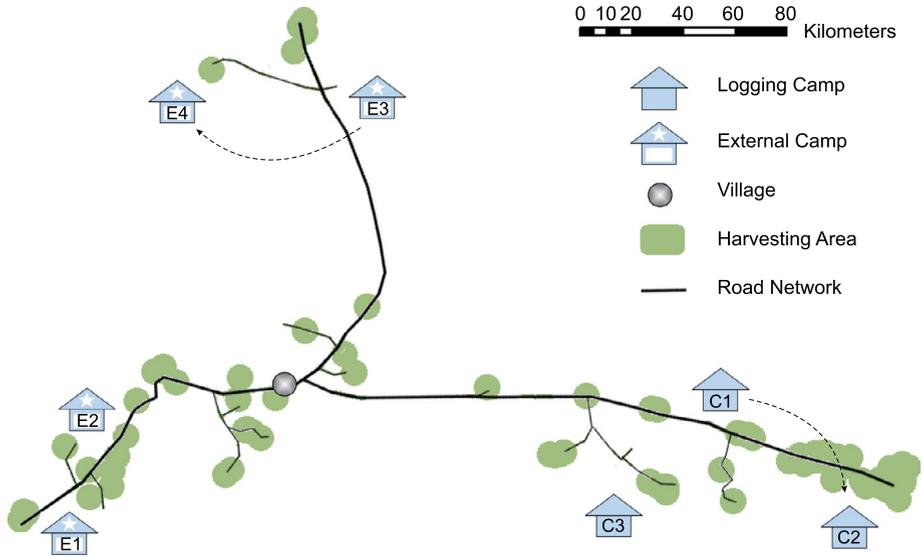
### 6.1 Comparative study for planning period 2006 to 2010

In this study, we simulate the activities performed by the company on two different levels: first, construction and relocation of logging camps and, second, the allocation of worker demand to accommodations. The results are then compared to the solution provided by the mathematical model.

#### 6.1.1 Data description

The company performed logging and road construction activities in an area which is divided into approximately 4000 different regions. These regions are geographically clustered to a total of 38 regions. The planning period starts at the summer season in 2006 and ends after the winter season in 2010. The logging and road construction activities were subject to significant variations throughout the seasons. The total average demands (in number of workers per day) for logging and road construction in each season are illustrated in Fig. 4. Note that the demands at each region are not necessarily clustered within subsequent seasons. Logging crews are formed by six workers, while road construction crews contain three workers. Demands for three logging supervisors and one road construction supervisor are estimated in proportion to the regions' work crew demands. All 38 working regions as well as the locations of the company's camps are available for potential camp construction or relocation. Detailed data for the entire road network, composed by roads categorized into four different conditions, are available. A simplified version is illustrated in Fig. 5. Logging and road construction regions are indicated by the green areas. Each road type allows a different vehicle average speed, such that transportation times and costs are computed fairly accurately. Costs take into account gasoline, vehicle renting and additional salary due to long travel times.

*Available accommodations and camp relocations* A village is located in a central location between the regions of forestry activities. According to the company, approximately two



**Fig. 5** Simplified illustration of the logging regions and the road network

out of five crews live in the village and may thus be hosted at zero costs. We thus roughly estimate that 40 % of the total worker demand may be hosted at the village, paying only the transportation costs. In addition, a practically unlimited number of hotel accommodations is available at the village for a price of 170\$ per person per night, including 54\$ for food.

In addition to the village, three logging camps from an external company are available. In the map, these camps are indicated by (capacity in parentheses in number of workers) E1 (65), E2 (40) and E3 (120). The latter has been relocated to location E4 after the winter season in 2008. External camps can be used on demand at an estimated price of 170\$ per person per night (food included).

The company itself held three camps in the beginning of 2006, indicated by C1 (60), C2 (96) and C3 (48). We assume that these camps hold trailers each with a capacity for twelve workers. After a few years, the location of camp C1 was too far from the new logging regions. Parts of this camp have thus been relocated to join the camp located at C2 after the winter season 2009, resulting in a larger camp for up to 144 workers. The costs for these camps involve significant economies of scale and thus depend on the size of the camp. The maintenance costs are around 1020\$ per day for a camp with a single trailer (capacity for twelve workers) and around 3400\$ for a camp with ten trailers (capacity for 120 workers). In addition, we assume a daily cost of 54\$ per worker for food. Maintaining large camps may thus be much cheaper than using the external camps and accommodations.

As can be recognized, the available capacities are very large compared to the total number of workers active in logging and road construction. This is due to the fact that other workers involved in forestry activities, such as forest management, tree planting, etc. use the camps. In addition, some capacities are used by the mining industry. However, the priority is always given to logging and road construction workers. We can thus assume that the entire capacity is available.

**Table 6** Cost distribution for the simulated company activities and the optimized solution

Costs (\$)	Simulated company activities	Optimized decisions
Hosting	2,683,588	2,770,588
Transportation	1,848,021	1,931,367
Maintenance	1,600,222	1,419,187
Trailer change	164,328	110,713
Sub-total	6,296,159	6,231,855
Construction	975,000	0
Relocation	302,470	0
Total	7,573,630	6,231,855

### 6.1.2 Comparison to the proposed planning

We now compare the activities performed by the company with the planning proposed by the mathematical model. As previously mentioned, we compare the planning decisions on two different levels:

1. Availability of capacities. We compare the decisions regarding camp construction and relocation.
2. Worker demand allocation. We compare the allocation of workers from working regions to accommodations.

*Decisions regarding camp construction and relocation* All optimization models are based on the CSLP model, defined by (12)–(36) and using the SAD inequalities (11) to facilitate the solution. We simulate the activities performed by the company by fixing all decisions regarding available capacities exactly as stated in Sect. 6.1.1. To be precise, we fix the relocation of external camp E3, the relocation of camp C1 and the construction of camp C3. As all construction and relocation decisions are fixed, the model is solved to optimality in a few minutes. The results for this scenario are compared to the optimal solution for a scenario where only the initial capacities of the company are fixed. External camp E3 is still relocated (as this is not a decision made by the company). However, the relocation of C1 and the construction of C3 are not fixed. The optimal solution for this scenario has been obtained within 20 hours of computation time (nine hours to prove optimality < 1 %). In contrast to the decisions made by the company, the optimal solution does not suggest any camp construction or relocation. Instead, the cheapest solution is obtained when using the available capacities at the same locations as found in the beginning of the planning. Table 6 summarizes the cost distribution for both scenarios. Without costs for construction and relocation, the costs for both scenarios are very similar. This suggests that, for the activities in the given time period, the initial locations of the existing camps, as well as their capacities were just as good as the locations and capacities achieved by the construction and relocation of camps. Adding the costs for construction and relocation to the costs for the company's activities results in a total cost that is much higher. Note that both scenarios assume optimal demand allocation. We will explore this topic further below.

*Decisions regarding the demand allocation* The previous analysis simulates the company's activities regarding the decisions of where to locate or relocate camps. For both scenarios, the results assume that the demand allocation is optimal, i.e., the amount of workers

**Table 7** Cost distribution for optimal and heuristic demand allocation

Costs (\$)	Optimal	Heuristic
Hosting	2,683,588	1,437,822
Transportation	1,848,021	3,093,429
Maintenance	1,600,222	2,424,474
Trailer change	164,328	186,881
Sub-total	6,296,159	7,142,606
Construction	975,000	975,000
Relocation	302,470	302,470
Total	7,573,630	8,420,076

from each region hosted at each accommodation, as well as the capacity level maintained at each camp during each season. As many cost factors have to be considered when allocating the workers to the accommodations, a manual allocation planning is likely to be far from optimal. Of course, many other factors may impact the decisions when allocating certain working regions to accommodations, such as the preferences of certain workers.

For the given planning period, the actual allocation of the workers to the available accommodations is not known. We thus use a simple heuristic to simulate the manual allocation planning. The allocation is performed for each season. We give priority to regions with large worker demands. Logging and road construction demands are thus considered in non-increasing order. We then select the accommodation that has the lowest cost for transportation and hosting. To compute the real cost, one should also consider the maintenance costs for open trailers at the company's camps. However, these costs depend on the actual occupation level of the camp and are thus difficult to estimate at the beginning of the heuristic planning. We thus do not include the maintenance costs in the total costs to emphasize the use of the company's own camps. Table 7 compares the cost distribution for the optimal and the heuristic demand allocation. From an economic point of view, the optimal demand allocation is 11.8 % cheaper when comparing the costs involved in hosting, transportation and camp/trailer maintenance. Note, however, that a planning as suggested in the optimal scenario is likely not to be completely feasible in practice. Many other factors may impact on such planning, such as the preferences of workers, changes in demand and other uncertainties. Workers will most likely prefer not to change their accommodation too often throughout a working season. An analysis of the suggested demand allocation in the optimal scenario shows that in most cases this criterion is met. Working crews are allocated to the same accommodation throughout the entire season. Only in a small number of cases, two different accommodations are suggested to host the same crew throughout a season. Although the allocation of workers to accommodations includes many other factors not considered in our analysis, the large cost difference found in our comparison emphasizes the difficulty of a cost efficient manual planning and suggests a potential to perform demand allocation in a more cost efficient manner, while respecting all other requirements.

## 6.2 Analysis of proposed planning for period starting in 2011

Based on the logging and road construction demands for the harvesting period 2011 to 2015, we now analyze the decisions proposed by the mathematical model.

**Table 8** Cost distribution in the optimal solutions for both scenarios

Costs (\$)	Scenario 1	Scenario 2
Hosting	1,879,905	1,476,083
Transportation	2,261,809	1,353,561
Maintenance	2,983,112	3,365,490
Trailer change	252,521	242,751
Construction	0	0
Relocation	0	302,470
Total	7,377,347	6,740,355

*Data description* The data contains 29 clusters of logging regions. The road network is similar to the one shown in Fig. 5. However, logging regions, as well as the locations of available accommodations, are different. The demands in this planning are much more balanced over the seasons than it was the case in the previous planning period. Demands require up to eight logging and four road construction crews. The complete demand is easily covered by five existing accommodations: the village and four camps (with 2, 3, 4 and 4 trailers, respectively). All other assumptions are similar to the ones made for the previous planning period.

*Solution analysis for different scenarios* Data about whether or not the company intends to construct or relocate camps were not available. We therefore do not compare to decisions of the company, but rather to two extreme scenarios to show how the proposed model may help in future decisions: one scenario where available capacities are not changed at all and one where capacities may be changed if beneficial. The first scenario thus considers only the existing accommodations at their original locations. Camp construction and relocation are thus not allowed. The second scenario assumes the original locations of the initially existing accommodations, but additionally allows the construction of new camps and the relocation of existing ones (once a year). Both scenarios are based on the CSLP model, defined by (12)–(36) and the SAD inequalities (11). Table 8 shows how costs are distributed in the optimal solution of each scenario. Scenario 2 suggests the relocation of a camp with four trailers after the fifth season. The additional camp relocation costs are outweighed by the savings in the transportation costs, which reduced by more than 40 %. This results in a very beneficial solution, reducing the total costs by 8.6 %.

Clearly, the reduction of the transportation costs is directly linked to the traveled time and distance. As can be seen in Table 9, the average distance traveled by the crews is reduced significantly (23 % and 16 %, respectively, in Scenario 2) when the camp is relocated. Finally, Table 9 also reports the proportion of time during which existing trailers are open. This percentage considers all existing trailers throughout all time periods. One can observe that slightly more trailers are opened in Scenario 2, i.e., the existing camps are better used than in the previous two scenarios. Maintenance costs increase, but lower transportation costs may be involved as such trailers are closer to certain logging regions.

## 7 Conclusions and future research

A mixed-integer programming model for the location of logging camps has been presented. This model extends the classical Capacitated Facility Location Problem by several features.

**Table 9** Usage of existing trailers and travel distances for the both scenarios

	Scenario 1	Scenario 2
Trailers open	49.4 %	54.8 %
<i>Average travel distance (km):</i>		
Logging crews	114	88
Road construction crews	129	109

Next to the well known features of multiple periods, multiple commodities and multiple capacity levels, further extensions include the partial and temporary closing of facilities, particular capacity constraints that include integer rounding and the integration of economies of scale on several levels of the cost structure. In addition, the model allows the extensions and relocation of existing facilities. Such integer rounding capacity constraints can be useful in other applications. As they increase the integrality gap and therefore the difficulty to solve the problem, new valid inequalities are derived to effectively reduce this integrality gap.

Instances based on a large variety of different properties have been generated. Experiments on these instances illustrated the impact of the different problem features on the difficulty to solve the problem. It is shown that general purpose solvers such as CPLEX are capable of solving most of the instances up to a realistic size in reasonable time, when using optimal solutions of a simplified problem as warm start solution for the entire problem. Case studies based on data from a Canadian logging company for two planning periods have been presented. The first study indicates a strong potential for economic savings on two different decision levels: where to locate the logging camps, as well as how to allocate worker demand from the working regions to the accommodations. The second study proposes a planning for the upcoming planning period of the company. It proposes the relocation of an existing camp, resulting in potential savings of more than 8 % of the total costs when compared to the scenario where camps stay at their current location.

Though most of the smaller and medium sized instances can be solved in reasonable time, some of the instances remain unsolved. The models for larger instances typically exceed the memory limitations of current standard computers, such as the ones used in the experiments. In order to solve these instances, more sophisticated solution techniques are necessary, such as mathematical decomposition. Interesting extensions of the model for future research include the possibility of partial relocation of camps, as well as the use of trailers of different sizes.

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